M.Sc. DEGREE EXAMINATION,

PART - II - FINAL

BRANCH - MATHEMATICS

Paper : I : FINITE MATHEMATICS AND GALOIS THEORY

(Revised Regulations from 2010-2011)

Max. Marks : 20

SECTION - A

Answer any **four** quesitons.Each question carries **5** marks.

1. Show that $s(r,n)=s(r-1,n-1)+ns(r-1,n)$ where $r,n\in N$ with $r\geq n$.

2. Find the coefficient of X^{R} $R\geq 13$ in the expansion of $\left(x^{3}+x^{4}+.....\right)^{6}$.

3. Prove that the degree sum of all vertices in any graph is even.

4. Prove that a graph and its compliment cannot both be disconnected.

5. If $f(x),g(x)\in Z[x]$ are primitive polynomials, then prove that their product $f(x)g(x)$ is also primitive.

6. Let F=Z/2. Then show that the splitting field $x^{3}+x^{2}+1\in F[x]$ is a finite field with eight elements.

7. If $f(x)\in F(x)$ has r distinct roots in its splitting field E over F, then prove that the Galois group $G(E/F)$ of $f(x)$ is a subgroup of the symmetric group S_r.

8. Show that the Galois group of $x^{4}+x^{2}+1$ is the same that of $x^{6}-1$ and is of order 2.

SECTION - B

Answer **One** questions from each **unit**. Each question carries **15** marks.

UNIT - I

9. a) Find the solution to $Y_{n+2}-4Y_{n+1}+4Y_{n}=3.2^{n}$

b) How many ways can 25 people be assigned to 3 different rows with at least one person in each row?

(OR)

10. a) Prove that the total number of k-permutations of a set A of n elements is given by $n(n-1)(n-2)\ldots\ldots.(n-k+1)$.

b) Let $A_i = \{\Pi \in S_n | \Pi(i) \in X_i\}$ be the bad permutation for i. Use principle of inclusion and exclusion, find the formula for $p(X_1\ldots\ldots\ldots.X_n)$.

UNIT - II

11. a) Prove that there are eleven nonisomorphic simple graphs on four vertices.

b) Prove that a graph is bipartite if an only if it contains no odd cycles.

(OR)

12. a) Let M be the incidence matrix and A be the adjacency matrix of a graph G. Then prove that every column sum of M is 2

b) Prove that in a tree any two vertices are connected by Unique path.

UNIT - III

13. a) State and prove Gauss Lemma.

b) Let p(x) be an irreducible polynomial in $F[x]$. Then prove that there exists an extension E of F in which $p(x)$ has a root.

(OR)

14. a) Let E be an extension of F. If K is the subset of E consisting of all the elements that are algebraic over F, then prove that K is a subfield of E and an algebraic extension of F.

b) Let $f(x) \in F(x)$ be a polynomial of degree ≥ 1 wit α as a root. Then prove that α is a multiple root if and only if $f(a) = 0$.

UNIT - IV

15. Let H be a finite subgroup of the group of automorphism of a field E. Then prove that $[E:E_H] = [H]$.

(OR)

16. Prove that every polynomial $f[x] \in C[x]$ factors into linear factors

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M.Sc. DEGREE EXAMINATION,

PART II - Final

BRANCH : MATHEMATICS

Paper - II : TOPOLOGY AND FUNCTIONAL ANALYSIS

(Revised Regulation from 2010-2011)

Max. Marks : 20

SECTION - A

Answer any **Four** questions. Each questions carries **5** marks.

1. Define a topological space. Let J_1 and J_2 be two topologies on a non empty set X. Then show that $J_1 \cap J_2$ is also a topology on X.
2. Prove that every sequentially compact metric space is compact.
3. Prove that in a Hausdorff space, any point and disjoint compact subspace can be separated by open sets, in the sense that they have disjoint neighborhoods.
4. Prove that the space $\mathbb{R}^n$ and $\mathbb{C}^n$ are connected.
5. If M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M, then prove that there exists a functional f_0 in N* such that $f_0(M) = 0$ and $f_0(x_0) \neq 0$.
6. If P is a projection on a Banach space B, and if M and N are its range and null space then prove that M and N are closed linear subspaces of B such that $B = M \oplus N$.
7. Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.
8. Prove that an operator T on a Hilbert space H is self - adjoint if and only if (Tx, x) is real for all x.

SECTION - B

Answer **one** question from each unit. Each question carries **15** marks.

UNIT - I

9. a. Let X be a topological space and A a subset of X. Let D(A) be the derived set of A. Then prove that

   i. $\overline{A} = A \cup D(A)$; and

   ii. A is closed $\Leftrightarrow A \supseteq D(A)$.

b. If f and g are continuous real or complex functions defined on a topological space X, then prove that $f+g, \alpha f, fg$ are also continuous.

(OR)

10. a. State and prove the generalized Heine - Borel theorem.

b. Prove that a closed subspace of a complete metric space is compact if and only if it is totally bounded.

UNIT - II

11. a. Prove that every compact Hausdorff space is normal.

b. State and prove Tietze extension theorem.

(OR)

12. a. Prove that a subspace of the real line $\mathbb{R}$ is connected if and only if it is an interval.

b. Let X be a compact Hausdorff space. Then prove that X is totally disconnected if and only if it has an open base whose sets are also closed.

UNIT - III

13. a. Let N and N' be normed linear spaces and T a linear transformation of N into N'. Then prove that the following conditions on T are all equivalent to one another :

i. T is continuous.

ii. T is continuous at the origin, in the sense that $x_n \to 0 \Rightarrow T(x_n) \to 0$;

iii. There exists a real number $k \geq 0$ with the property that $\|T(x)\| \leq k\|x\|$ for every x in N;

iv. If $S = \{x : \|x\| \leq 1\}$ is the closed unit sphere in N, then its image T(S) is a bounded set in N'.

b. Let B and B' be Banach spaces. If T is a continuous linear transformation of B onto B', then prove that image of each open sphere centered on the origin in B contains an open sphere centred on the origin in B'.

(OR)

14. a. Let B be a Banach space, and let M and N be closed linear subspaces of B such that $B = M \oplus N$. If z=x+y is the unique representation of a vector in B as a sum of vectors in M and N, then prove that the mapping P defined by $P(z) = x$ is a projection on B whose range and null spaces are M and N respectively.

b. Let B be a Banach space and N a normed linear space if $\{T_n\}$ is a sequence in B (B,N) such that $T(x) = \lim T_n(x)$ exists for each x in B then prove tht T is a continuous linear transformation.

UNIT - IV

15. a. Let B be a complex Banach space whose norm obeys the parallelogram law, and if an inner product is defined on B by $4(x, y) = \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2$, then prove that B is a Hilbert space.

b. If M is a closed linear subspace of a Hilbert space H, then prove that $H = M \oplus M^{\perp}$.

(OR)

16. a. Let H be a Hilbert space - prove that the adjoint operation $T \to T^*$ on B(H) has the following properties :

i. $(T_1 + T_2)^* = T_1^* + T_2^*$.

ii. $(\alpha T)^* = \overline{\alpha} T^*$

iii. $(T_1 T_2)^* = T_2^* T_1^*$

iv. $T^{**} = T$

v. $\|T^*\| = \|T\|$

vi. $\|T^* T\| = \|T\|^2$.

b. If T is an operator on a Hilbert space H, then prove that the following conditions are all equivalent to one another :

i. T*T=I

ii. (Tx,Ty)=(x,y) for all x,y;

iii. $\|Tx\| = \|x\|$ for all x.

M.Sc. DEGREE EXAMINATION,

PART II - FINAL

BRANCH : MATHEMATICS

Paper - III : OPERATIONS RESEARCH

(Revised Regulation from 2010-2011)

Max. Marks : 20

SECTION - A

Answer any **Four** questions. Each question carries **5** marks.

1. Define the term feasible solution and reduce the L.P.P. into its standard form :

   Minimize $Z=-3x_1+x_2+x_3$

   Subject to $\begin{array}{l} x_1-2x_2+x_3\le 11 \\ -4x_1+x_2+2x_3\ge 3, \\ 2x_1-x_3=-1;\ x_1,x_2\ge 0 \end{array}$

   x_3 unrestricted.

2. If $x_1=2, x_2=4$ and $x_3=1$ is a feasible solution of the system of equations $2x_1-x_2+2x_3=2$, $x_1+4x_2=18$

   Reduce the given feasible solution to a basic feasible solution.

3. Write the dual of the L.P.P.

   Minimize $Z=4x_1+6x_2+18x_3$

   Subject to $\begin{array}{l} x_1+3x_2\ge 3 \\ x_2+2x_3\ge 5 \\ x_1,x_2,x_3\ge 0 \end{array}$

4. Find an initial basic feasible solution of the transportation problem using the north west corner rule

				Supply
5	3	6	2	19
4	7	9	1	37
3	4	7	5	34
Demand 16	18	31	25	

5. Explain the formulation of Travelling salesman problem as Assignment problem.

6. Explain the fundamental EOQ problem.

7. Write two relations between average Queue length and average waiting time.

8. Discuss the probability distributions in Queueing systems.

SECTION - B

Answer **one** question from each unit. Each question carries **15** marks.

UNIT - I

9. a. Using graphical method find the maximum value of $Z = 10x_1 + 6x_2$

Subject to $5x_1 + 3x_2 \leq 30$
$x_1 + 2x_2 \leq 18$
$x_1, x_2 \geq 0$

b. Use simplex method to solve

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

Subject to $x_1 + 2x_2 + x_3 \leq 430$
$3x_1 + 2x_3 \leq 460$
$x_1 + 4x_3 \leq 420;$
$x_1, x_2, x_3 \geq 0$

(OR)

10. a. Show that dual of a dual is the primal.

b. State and prove fundamental theorem of Duality.

UNIT - II

11. Explain the least cost method and solve the transportation problem

				Supply
11	13	17	14	250
16	18	14	10	300
21	24	13	10	400
Demand 200	225	275	250	

(OR)

12. a. Explain the vogel's approximation method.

b. Solve the assignment problem

10	25	15	20
15	30	5	15
35	20	12	24
17	25	24	20

UNIT - III

13. Use dynamic programming to solve the problem

Minimize $Z = y_1^2 + y_2^2 + y_3^2$

Subject to $y_1 + y_2 + y_3 \geq 15,\ y_1, y_2, y_3 \geq 0$.

(OR)

14. If a project has the following time schedule given as

Activity	Time in weeks	Activity	Time in weeks
1-2	2	4-6	3
1-3	2	5-8	1
1-4	1	6-9	5
2-5	4	7-8	4
3-6	8	8-9	3
3-7	5		

Find the total float and critical path.

UNIT - IV

15. a. Write the operating characteristics of a Queueing system.

    b. The rate of arrival of customers at a public telephone booth follow poisson distribution with an average time of 10 minutes between customer and the next. The duration of a phone call is exponential distribution with near time of 3 minutes then find

    i. The probability that a person arriving at the booth will have to wait.

    ii. Average length of non - empty queues.

(OR)

16. Explain the EOQ problem with finite Replenishment and write its characteristics.

M.Sc. DEGREE EXAMINATION,

PART II - Final

BRANCH : MATHEMATICS

Paper - IV : NUMBER THEORY

(Revised Regulation from 1999-2000)

Max. Marks : 20

SECTION - A

Answer any **Four** questions. Each questions carries **5** marks.

1. Prove that $\sum_{d/n} \phi(d) = n$.

2. State Euler - Fermat theorem.

3. Define

   i. Chebyshev's ψ - function and

   ii. Chebyshev's θ - function.

4. Define character of a finite group G.

5. Let g be any real valued character mod k and let $A(n) = \sum_{d/n} y(d)$. Then prove that $A(n) \geq 0$ for all $n \geq 1$ and $A(n) \geq 1$ if n is a square.

6. Define

   i. n is a quadratic residue modulo a prime p.

   ii. n is note a quadratic residue modulo a prime p.

7. Let a and m be relativity prime integers. When do you say that a is a primitive root modulo m.

8. For a fixed $k \geq 1$, let $g(n) = \sum_{m=0}^{k-1} e^{\frac{2\pi i m^n}{k}}$. Then prove that $g(n) = \begin{cases} 0 \text{ if } k \nmid n \\ k \text{ if } k / n \end{cases}$.

SECTION - B

Answer **one** question from each unit. Each question carries **15** marks.

UNIT - I

9. a. Let f be multiplicative function. Then, prove that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n).f(n)$ for all $n \geq 1$.

b. State and prove Euler's summation formula.

(OR)

10. a. Assume that $(a,m) = d$. Then prove that the linear congruence $ax \equiv b(\text{mod } m)$ has solutions if and only if d/b.

b. State and prove chinese remainder theorem.

UNIT - II

11. For $n \geq L$, prove that $\frac{1}{6}.\frac{n}{\log n} < \pi(n) < 6.\frac{n}{\log n}$.

(OR)

12. State and prove Selberg's asymptotic formula.

UNIT - III

13. Prove that a finite abelian group of order n - has exactly n - distinct characters.

(OR)

14. If the relation $\pi_a(x) \sim \frac{\pi(x)}{\phi(k)}$ as $x \to \infty$ holds for every integer 'a' relatively prime to k then prove that $\pi_a(x) \sim \pi_b(x)$ as $x \to \infty$ whenever $(a,k) = (b,k) = 1$.

UNIT - IV

15. Let $S_k(n) = \sum_{d/(n,k)} f(d)g\left(\frac{k}{d}\right)$ where f and g are multiplicative. Then prove that $s_{mk}(ab) = s_m(a).s_k(b)$ whenever (a,k)=(b,m)=1.

(OR)

16. a. Let (a,m) = 1. Then prove that a is a primitive root mod m if and only if the numbers $a, a^2, \ldots, a^{\phi(m)}$ form a reduced residue system modulo m.

M.Sc. DEGREE EXAMINATION,

PART - II : FINAL

BRANCH : MATHEMATICS

Paper -V - MATHEMATICAL STATISTICS

(Revised Regulation from 2010-2011)

Max. Marks : 20

SECTION - A

Answer any FOUR questions. Each questioncarries 5 marks.

1. Explain

   i) Random variable.

   ii) Stochastic Independence.

2. Let the random variable X have the pdf where $f(x)=\begin{cases} 2x & ;0<x<1 \\ 0 & ;elsewhere \end{cases}$.

   Find the Probability density function?

3. Illustrate Gamma and Chi-Square distribution

4. A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each colour?

5. If X has the moment generating function $M(t)=e^{2t+32t^2}$ prove that X has a normal distribution with M=2 and $\sigma^2=64$.

6. Explain MLE of θ in Estimation.

7. Define

   a) Testing of Hypothesis

   b) Likelihood ratio test

8. Let $X_1, X_2, X_3, \ldots X_n$ be a random sample from $U[0, \theta]$ population. Obtain MVUE for θ.

SECTION - B

Answer ONE question form each Unit. Each question carries 15 marks.

UNIT - I

9. a) State and prove Chebyshev's Inequality.

b) Given the joint density function of X and Y as

$$f(x,y)=\begin{cases}\frac{1}{2}x\exp(-4) & ; \quad 0<x<2, y>0\\ 0 & ; \quad elsewhere\end{cases}$$

Find the distribution of X+Y?

(OR)

10. a) State and prove Baye's theorem.

b) Let X and Y have the Joint probability density function described as follows :

(x,y)	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
f(x,y)	2/15	4/15	3/15	1/15	1/15	4/15

and $f(x, y)$ is equal to zero elsewhere. Find the Correlation Coefficient ρ?

UNIT - II

11. Derive the Poisson distribution as a limiting case of Binomial distribution.

(OR)

12. Fit the Binomial distribution for the following data and Compare the theoretical frequency with actual one.

X	0	1	2	3	4	5
Frequency	2	14	20	34	22	8

UNIT - III

13. Explain Normal distribution and its properties.

(OR)

14. State and explain t and F distributions.

UNIT - IV

15. State and prove Neyman-Pearson Lemma

(OR)

16. Define Cramer-rao Inequality
